NUMBER SENSE

9-12

Number Sense
What All Students "Should Know"
9 – 12

1. Addition, subtraction, multiplication, and division with real numbers.

Clarifications:
All students should be able to identify and use:

- Examples of real numbers 0, 2, -3, ½, .6, .2121, $4\frac{1}{3}$, $\sqrt{7}$, Π.

Note: Students should also be aware of non-real numbers (imaginary and complex)

- Examples of imaginary numbers *i*, 3*i*, -5*i*
- Examples of complex numbers 1 + 2*i*, 3 - 5*i*

2. Numbers and their relationships can be represented in multiple forms.

Clarifications:
All students should recognize:

- Numbers in multiple forms for R (real numbers) such as:
  - ♦ Integers $5 = \frac{10}{2} = \sqrt{25} = 5.0$
  - ♦ Fractions $\frac{1}{2} = \frac{2}{4} = \frac{13}{26}$
  - ♦ Decimals $5.3 = \frac{53}{10} = 5\frac{3}{10}$
  - ♦ Percents $50\% = .50 = \frac{50}{100}$
  - ♦ Exponential form 2^3, $\sqrt{2^6}$
  - ♦ Scientific notation 5.2×10^4

- Relations can be shown in multiple forms such as:
  - ♦ Tables
  - ♦ Verbal rules
  - ♦ Equations
  - ♦ Inequalities
  - ♦ Graphs

x	0	1	2	3
y	2	5	8	11

The results are obtained by multiplying your number by 3 then adding 2.
$y = 3x + 2$

3. The appropriate use of technology.

Clarifications:
All students should have ready access to technology, specifically graphing calculators and computers. Appropriate use of these tools allow students to explore concepts, validate conjectures, manipulate large numbers, analyze large sets of data, and graph equations and data. Only through the use of these tools, in addition to paper/pencil, mental math and estimation, will students understand which tool is best for them to use in solving the given problem.

NOTE: Please see the technology statement in the Curriculum Frameworks on page vii.

4. Data can be organized in many forms.

Clarifications:
All students should know different forms for organizing data including:
- Various diagrams
- Graphs
- Tables
- Charts
- Calculator/computer lists
- Spreadsheets

See examples on Benchmark 2.

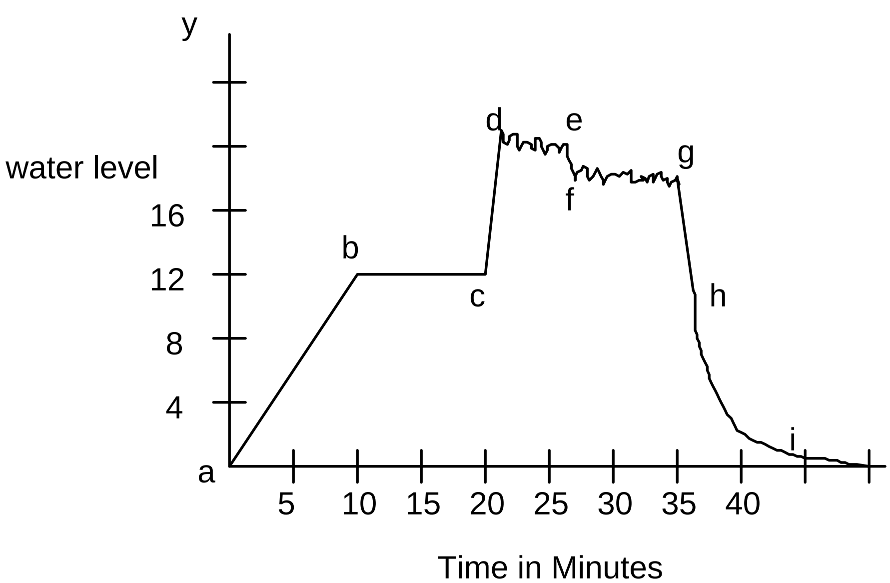
Number Sense
What All Students "Should Do"
9 – 12

Written Benchmark: A

Develop, analyze, and explain procedures used for representing and analyzing relationships in tables, verbal rules, equations, and graphs.

<u>Problem 1</u>:

Process Standards: 2.4 and 3.5



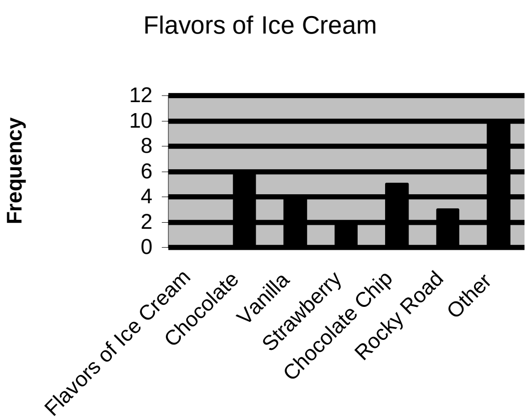
Write a story that describes the change in water level in a wading pool over time. Your story should be in paragraph (not outline) form and should explain the changes that occur at each of the marked locations.

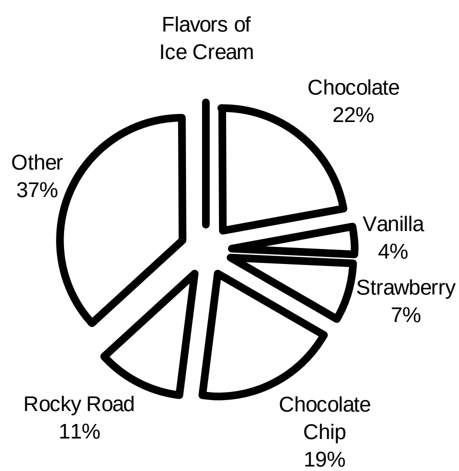
Sample story:
After putting out their wading pool, Sally begins to fill it with the hose. The water is filling the pool at a constant rate. About 10 minutes later, the pool is full and Sally turns off the hose. She then goes into the house to get the kids ready to "swim." Ten minutes later the kids come running out of the house and all jump into the pool at once causing the water level to quickly rise! The kids play in the water causing the water level to rise and fall as they splash around. All of the sudden, one child slips and falls into the water creating a huge wave that sloshes water over the edge of the pool. With everyone okay the kids continue to splash around for another eight minutes then all hop out of the pool together. Sally then dumps the water onto her flowerbed and puts the pool away leaving little droplets to dry later in the day.

<u>Problem 2</u>:
Process Standards: 1.8 and 3.3

Mr. Jones surveyed his Math students on their favorite flavors of ice-cream. The following chart, table, and graphs show the results:

Flavor of Ice Cream	**Number of Students**
Chocolate	***6***
Vanilla	***4***
Strawberry	***2***
Chocolate Chip	***5***
Rocky Road	***3***
Other	***10***





A. National surveys indicate that vanilla is still the most popular flavor. How do the survey data compare with the national trends?

B. Which representation of data was most helpful? Explain your answer.

Solution Notes:
Answers may vary, but explanations should clarify students' positions.

Prerequisites:
Students should:
1. Have a basic understanding of graphs and know how to draw different types of graphs.

Problem 3:
Process Standards: 1.6 and 3.5

Examine the table below. Find the missing values so that all of the points are collinear. Explain how you found these values.

x	y
3	5
4	y
7	13
x	1

Solution Notes:
x = 1. For the points to be on the same line, $\frac{\text{y-change}}{\text{x-change}}$ must be the same for any two points.

Since $\frac{5-13}{3-7} = \frac{-8}{-4} = 2$, then $\frac{5-1}{3-x} = 2$ and $\frac{y-13}{4-7} = 2$.

so 4 = 6 – 2x
-2 = -2x
x = 1

y – 13 = -6
y = 7

Prerequisites:
Students should:
1. Know the meaning of collinear.
2. Be able to discuss rates of change.
3. Apply patterns to linear situations.

Students should be able to discuss that for each change of 1 in x, y would increase by 2. So, when x goes from 3 to 4, y should go from 5 to 7. If y goes down from 13 to 1, then x should go down 6 from 7 to 1.

Number Sense
What All Students "Should Do"
9 – 12

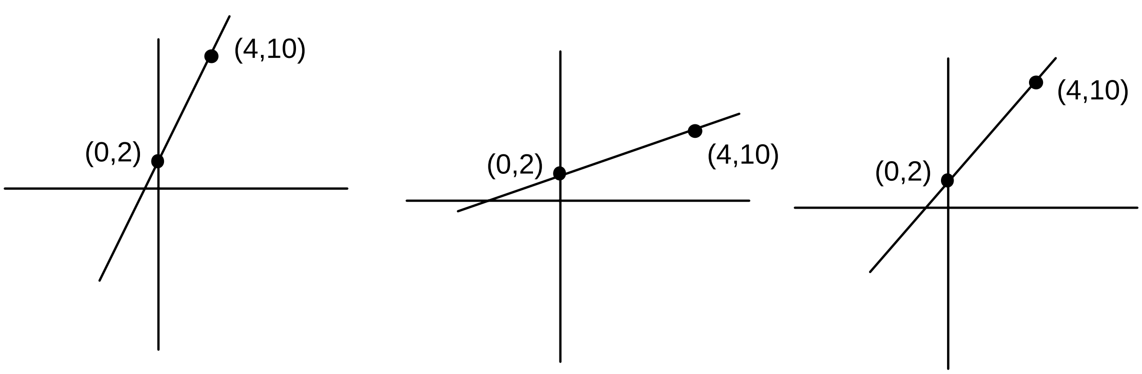
Written Benchmark: B
Analyze the effects of parameter changes
on the graphs of functions.

<u>Problem 1</u>:
Process Standards: 1.6

Answer the following questions about the three graphs shown below:

1. Explain why the three graphs must be variations of the same line.

2. Explain how, if they are the same line, the three lines could appear to have different slopes?



Solution Notes:
Student answers may vary but should include justifications as to why lines are equivalent (e.g. generating equations of lines given two points) and should demonstrate understanding of impacts of changing axes scale values.

Prerequisites:
All students should be able to:
1. Graph a line or use a graphing calculator to graph a line.

Problem 2:
Process Standards: 1.6 and 3.5

Using either graph paper and pencil or a graphing calculator, complete the following:

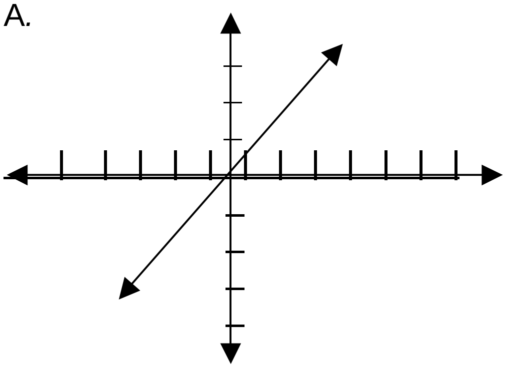
A. Graph the line y = x

B. What happens to the graph of y = x when it is changed to y = x + C, and C is positive? How does the graph change, when C is negative?

C. What happens to the graph of y = x when it is changed to y = Ax, when A > 1? What happens to the graph of y = Ax when 0 < A < 1? What happens to the graph when A is negative?

Solution Notes:

A.



B. The line shifts up when C is positive. The line shifts down when C is negative.

C. The line shift when A > 1 is steeper (rising from left to right) than the graph y = x. The line shift when 0< A <1 is not as steep as y = x. When A is negative then the line goes down from left to right.

Prerequisites:
Students should:
1. Know how to graph a line using pencil and paper or with a graphing calculator.
2. Know about the line's intercepts with the axes.
3. Know the concepts of slope and rate of change.
4. Know and be able to use the relationships of transformations (i.e. the impact of changing slope or intercept values).

<u>Problem 3</u>:
Process Standards: 1.4 and 3.5

A. Graph $y = x^2$ on a graphing calculator. Show a sketch of the graph on your paper.

B. On the same coordinate plane graph $y = 3x^2$. Analyze and state the transformation between $y = x^2$ and $y = 3x^2$.

C. On the same coordinate plane, graph $y = 3x^2 + 5$. Analyze and state the transformation between $y = x^2$, $y = 3x^2$ and $y = 3x^2 + 5$.

D. Indicate the window you selected to graph these equations and explain your selection.

Solution Notes:

A. Use a graphing calculator screen with a window of (-5, 5) (-2, 12). (Insert the display of the graph from the graphing calculator.)

B. A vertical stretch (by a factor of 3).

C. Shift $y = 3x^2$ up 5 units. Vertically stretch $y = x^2$ (by a factor of 3) and then shift up 5 units.

D. Answers will vary, but should include the domain and range values necessary to see all the graphs they are comparing.

Number Sense
What All Students "Should Do"
9 – 12

Written Benchmark: C

Analyze and describe relationships and the resulting effects between changes in an independent variable and a dependent variable.

<u>Problem 1</u>:
Process Standards: 1.10

The data shown below compares the braking distance needed to stop a car at the given speed.

Speed (MPH)	Distance (Nearest feet)
20	15
30	33
40	59
50	93
60	133

A. Do these variables (speed and distance) relate to each other in a linear fashion? Explain your answer. (You may graph if you wish)

B. Would the formula $b = .037V^2$ (where *b* is the braking distance and *V* is the speed in miles per hour) be a reasonable approximation for the data above? Explain your answer.

C. Compare the braking distance needed for 65 mph to that of 70 mph. How do the results relate to the advice to allow a two-car length space when following another car?

Prerequisites:
All students should know:

1. How to evaluate the changes in a relationship.
2. How to identify and use the independent variable.
3. How to identify and use the dependent variable.
4. How to identify and use the Domain and Range.

<u>Problem 2</u>:
Process Standards: 1.6

Independent Variable	**Dependent Variable**
4	2
25	5
36	6
40	$\sqrt{40}$

A. What term describes the relationship shown between the Independent Variable and the Dependent Variable?

B. As the Independent Variable gets larger, what happens to the Dependent Variable?

C. Given the data provided, one might believe the Independent Variable is always larger than the Dependent Variable. For what domain is this true? For what domain is this statement false? Justify your answers.

D. Graph the data provided and any other pieces of data you have generated for this problem. Justify your answers. Can the Dependent Variable ever be negative? Can the Independent Variable ever be negative?

Solution Notes:

A. Students should indicate that the Dependent Variable is the square root of the Independent Variable.
B. It increases also.
C. True for all $x > 1$, false for x between 0 and 1. Justifications can refer to specific examples such as $x = 1/4$, and $x = 1$. They could also refer to the graph of the function in their justification.
D. Students should indicate that since this is a function, the Dependent Variable could never be negative. Furthermore, the Independent Variable cannot be negative in the real number system, because the square root of a negative number is undefined. However, if the domain includes imaginary or complex numbers, then the Independent Variable could also be negative.